Good evening!

Thank you for coming.

This evening we will provide you with information on how we teach Mathematics at Moorlands and suggest ways that you can help your child at home using similar methods.

Maths at Moorlands.

A focus on Concrete, Pictorial and Abstract approaches. (CPA)

Karen Chapman and Justine Jackson

Maths Leaders Moorlands Primary School

Why are we engaging parents?

BBC News Report 2006

69% of parents do not help children with their homework because…

*Everything has changed since they were at school and they are not **confident** in the new methods.*

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Parent confidence and support

Lots of initiatives have been introduced, like ‘Keeping up with the children,’ ‘Inspire workshops’ and employing parental engagement coordinators in some schools.

Despite this, the situation worsened:

BBC News Report 2010

82% of parents feel unable to help pupils with their homework.

Lack of confidence.

The ‘problem’ with maths

MATHS HUBS
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“My dad thinks that the way **he** does maths is easier and better than **my** way but he doesn’t understand my way and his way confuses me.”

Pupil – Catford High School

In the Impact in Learning maths programme, children regularly talked about the **clash** between the maths learnt in school and what parents were showing them at home.

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Why it is important to engage parents with the mathematical learning of their children?

Research evidence suggests that when parents are engaged in their children's learning, outcomes for children can be improved.

Research also highlights the fact that parents feel they need more support to understand the current curriculum content and how they can support their child with their learning at home.

Desforges, C. and Abouchaar, A. (2003); Goodall, J. and Vorhaus, J. (2011); The Education Endowment Foundation (2019); Sarjeant, S. (2021)

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Agenda

- The theory behind the importance of CPA
- Using manipulatives to introduce the basics of a new concept and ways to replicate this at home.
- Transitioning between concrete, pictorial and abstract.

Concrete, Pictorial and Abstract Methods

Importance of CPA

In his research on the cognitive development of children (1966), Jerome Bruner proposed three ways of working to aid development:

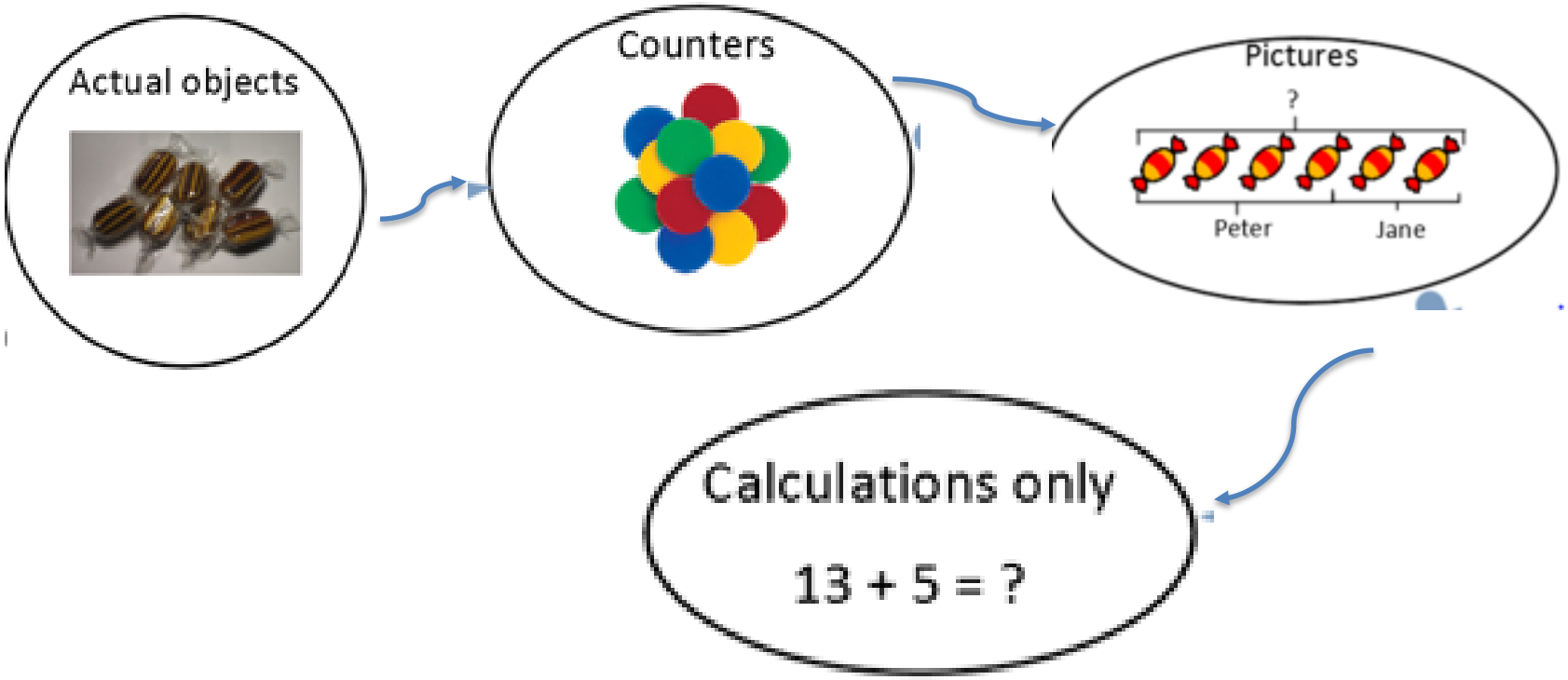
- Enactive representation (using 'concrete' objects)
- Iconic representation (drawing images / pictures)
- Symbolic representation (abstract numbers)

"If we do not use concrete manipulations, then we can not understand mathematics. If we only use concrete manipulations, then we are not doing mathematics."

Gu (2015)

Concrete, Pictorial, Abstract

Children should work at the stage they need until ready to move on. Within a class children can be working on the same calculation but accessing it in different ways.



Using CPA methods

Today we aim to give you a quick insight into methods used at school for the 4 rules of number and how you could adapt those to work at home, with a focus on:

- *Place value*
- *Addition and Subtraction*
- *Multiplication and Division*
- *Possible scope for a fractions evening later in the year if people would like that.*

Place Value

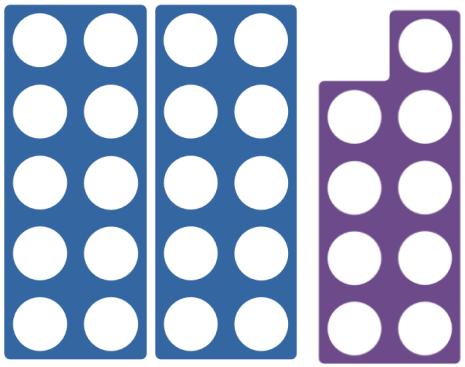
(understanding the value of each digit and it's place in the number system)

Place value

How many ways can you show 29?

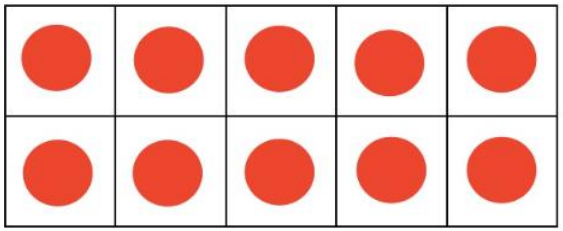
Twenty nine

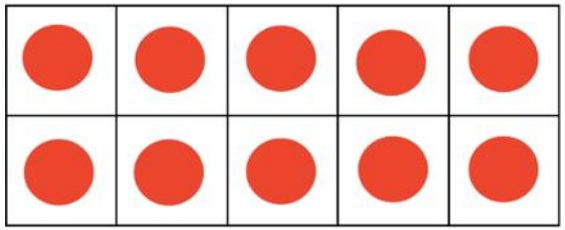
20 + 9

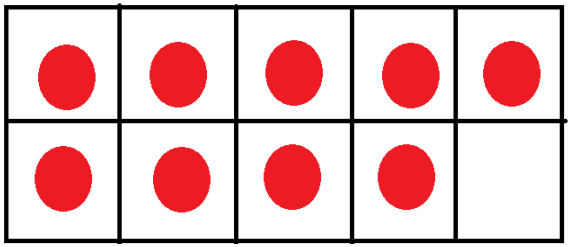


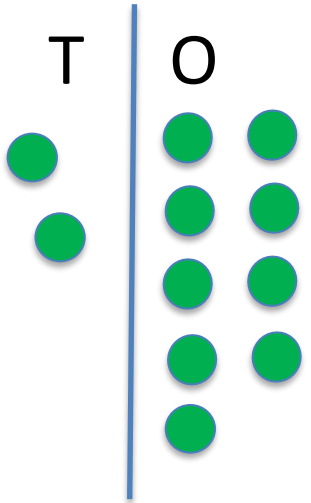
29

10 + 19



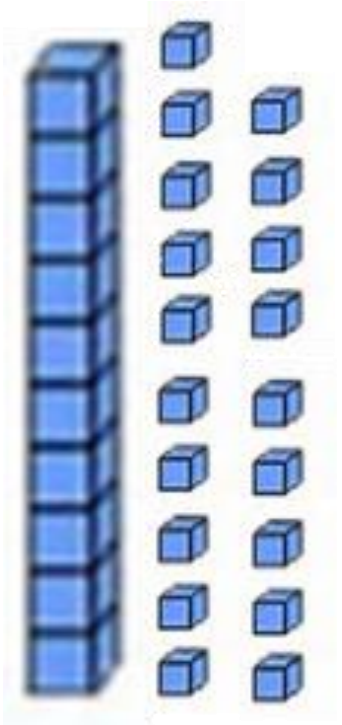






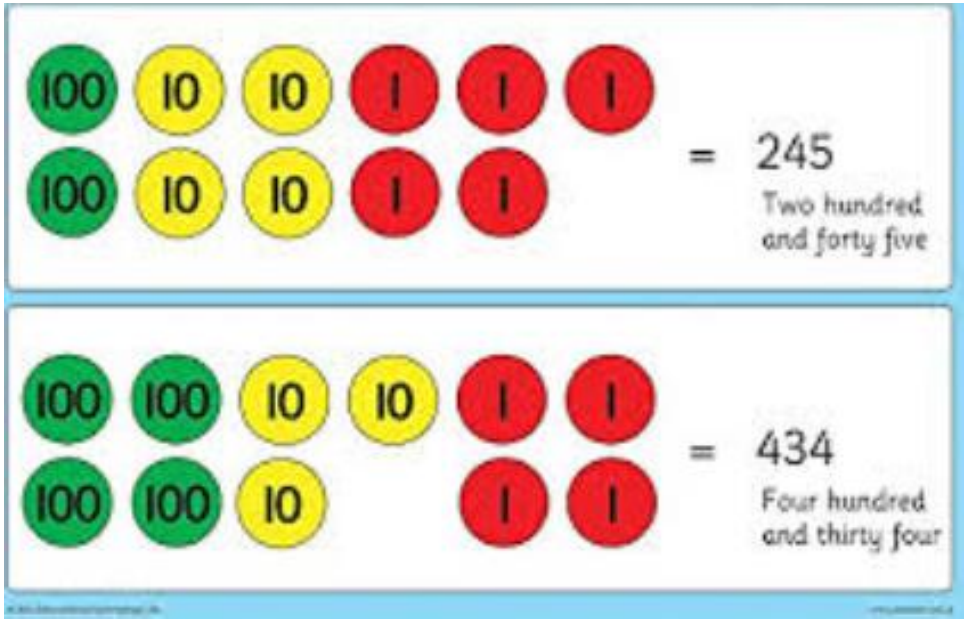
10 + 10 + 9

In KS1 children will work with base 10 resources when working with 2 digit numbers. This equipment helps them 'see' the 'ten-ness' of ten.



The next step…

- This moves on to Place Value Counters in KS2. (Year 3 / 4 / 5/ 6)



Pictorial Place Value counters

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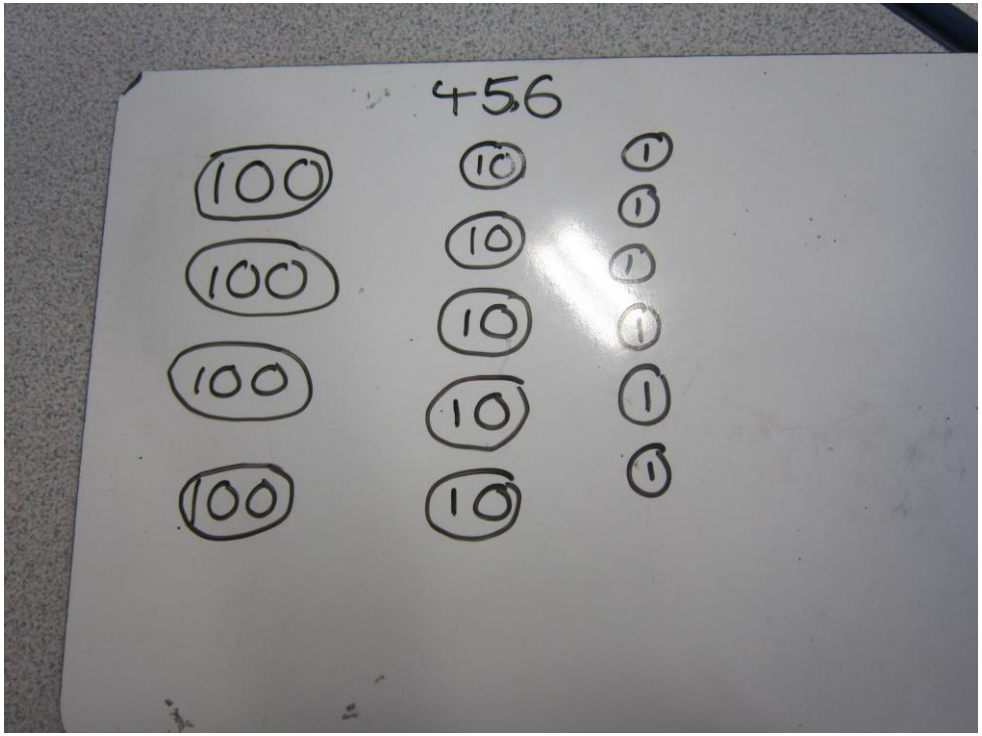
Draw the counters

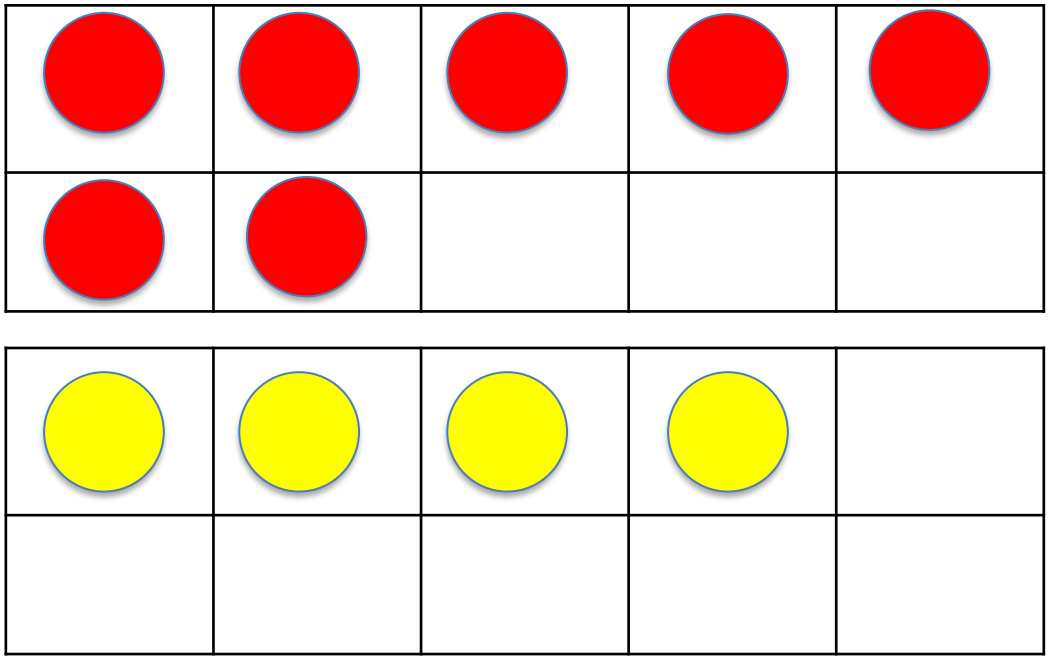


Addition

KS1 Addition- Regrouping to make 10

Solve… 7 + 4

Model



Calculations

7 + 4 = 11

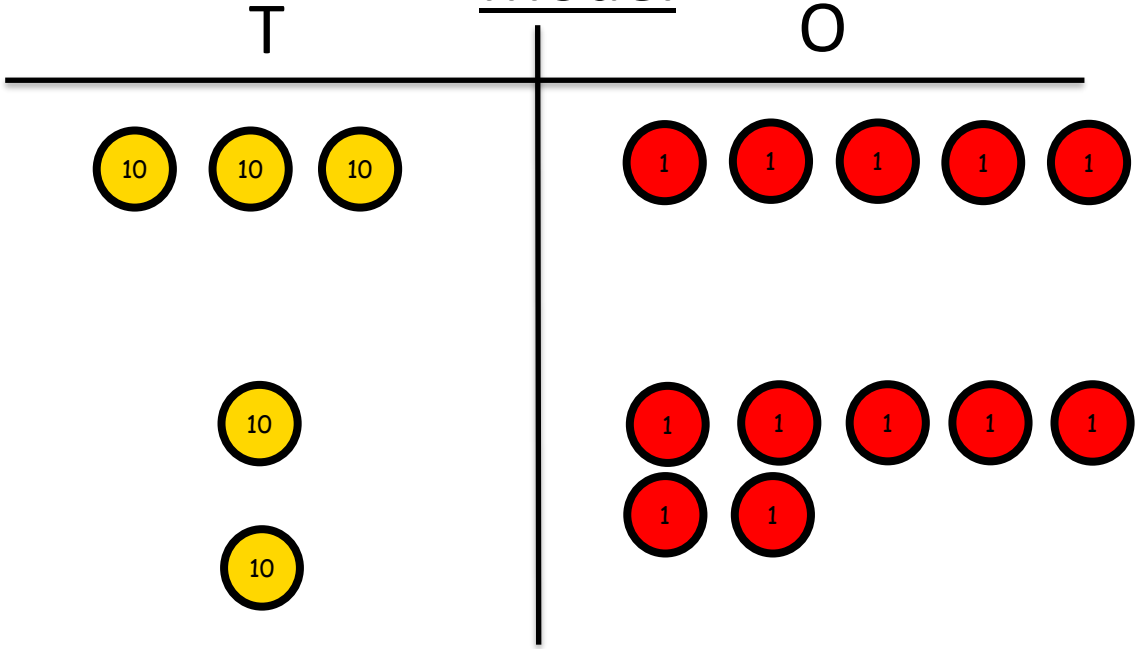
Beginning to use formal written methods

- Formal column method will first of all be introduced using the visual of place value counters. This is so that children fully understand what is happening with the numbers when we exchange and carry over rather than just learning a process.

Addition- Column method

Solve… 35 + 17 =

Model



Calculations

$$\begin{array}{r} 35 \\ +17 \\ \hline 52 \\ \hline \end{array}$$

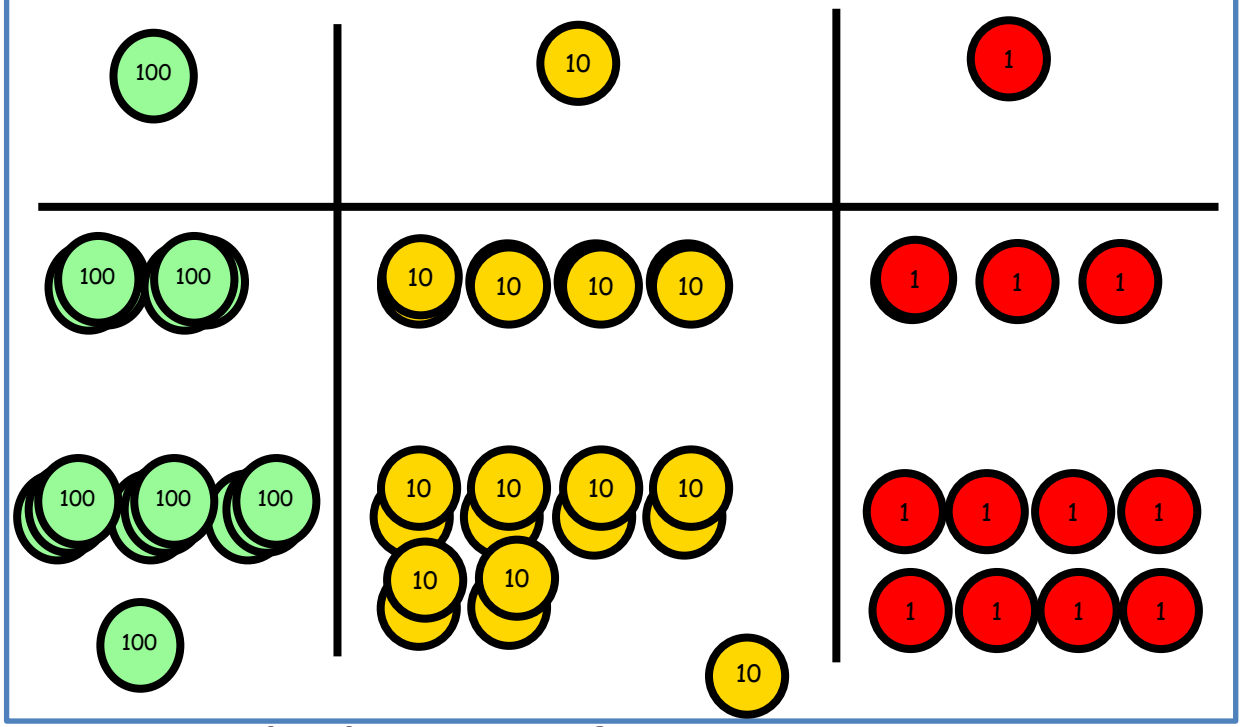
Key vocabulary: **exchange**

Can we exchange any counters?

Addition- Column method

Solve… 243 + 368 =

Model



Calculations

243

+368

1 1

6 11

Key vocabulary: **exchange**

Can we exchange any counters?

Moving to pictorial

After lots of experience with the actual counters children should be able to draw the place value counters to help them solve a calculation, crossing out any counters that are to be exchanged.

Abstract

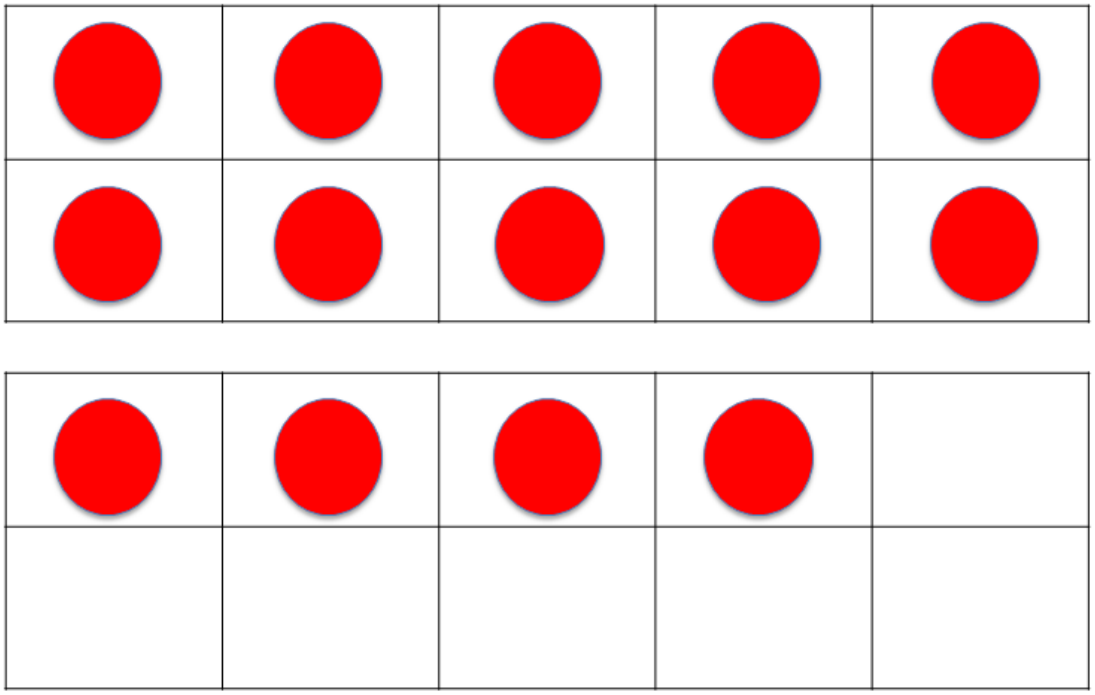
Children will naturally move away from drawing counters and carry out the calculation abstractly. Here it is clear the importance of being able to add single digits together with ease, including crossing the ten, as we looked at on the earlier slide. The highest numbers they will ever have to add is 9+9

Subtraction

Subtraction - make 10

Solve… 14 – 5 =

Model



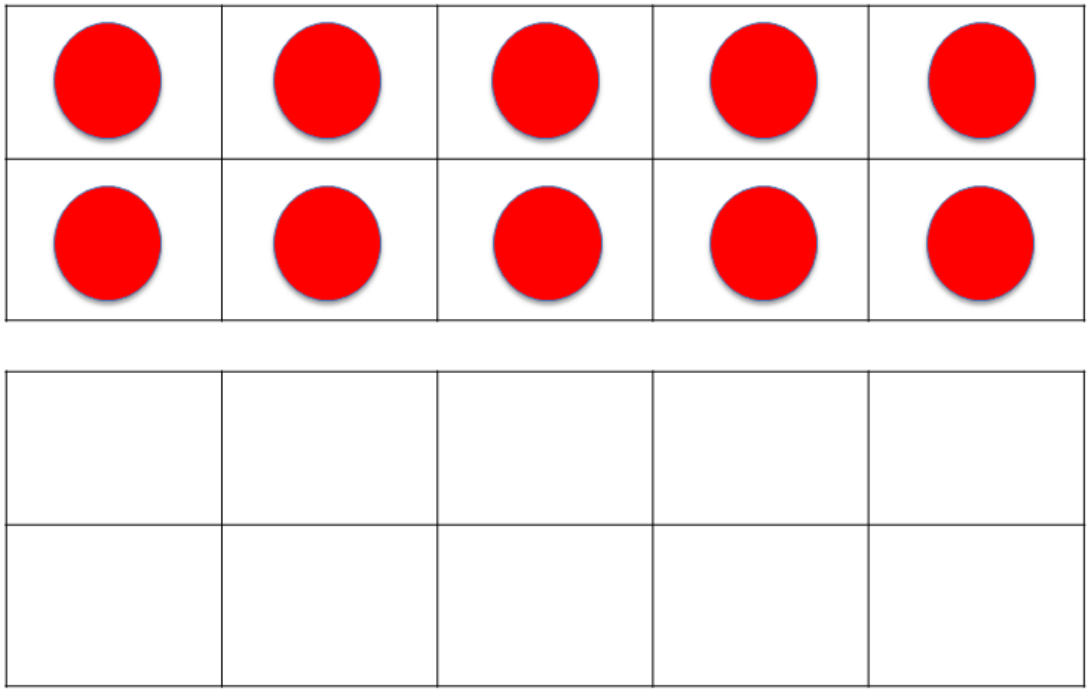
Calculations

14 – 5 =

Subtraction - make 10

Solve… 14 – 5 =

Model



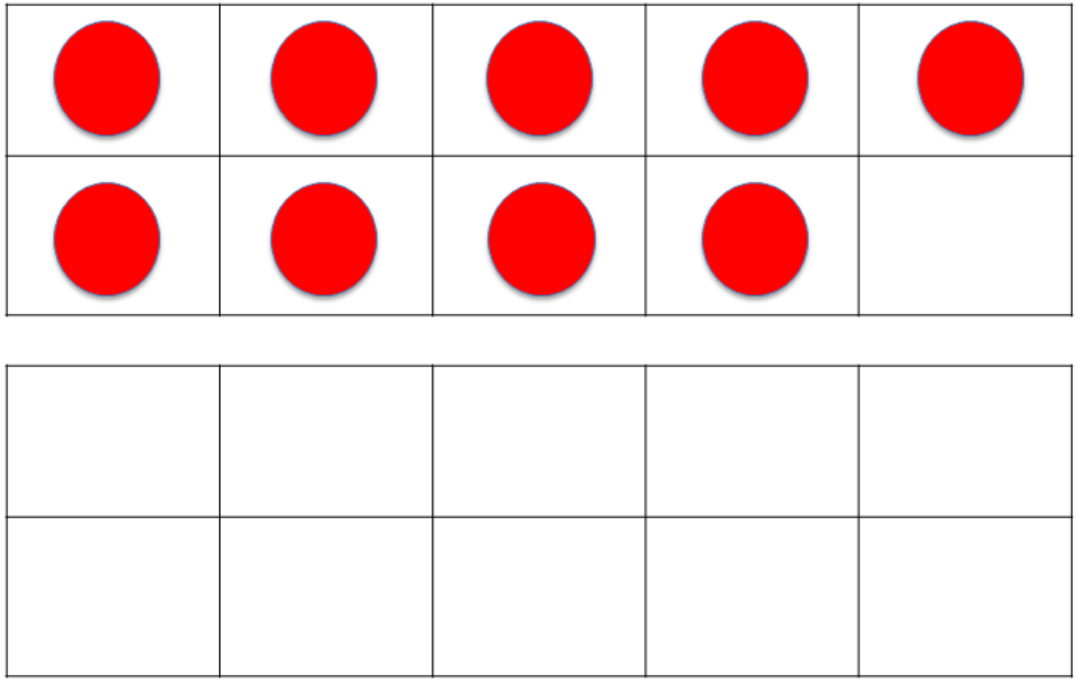
Calculations

14 – 5 =

Subtraction - make 10

Solve… 14 – 5 =

Model



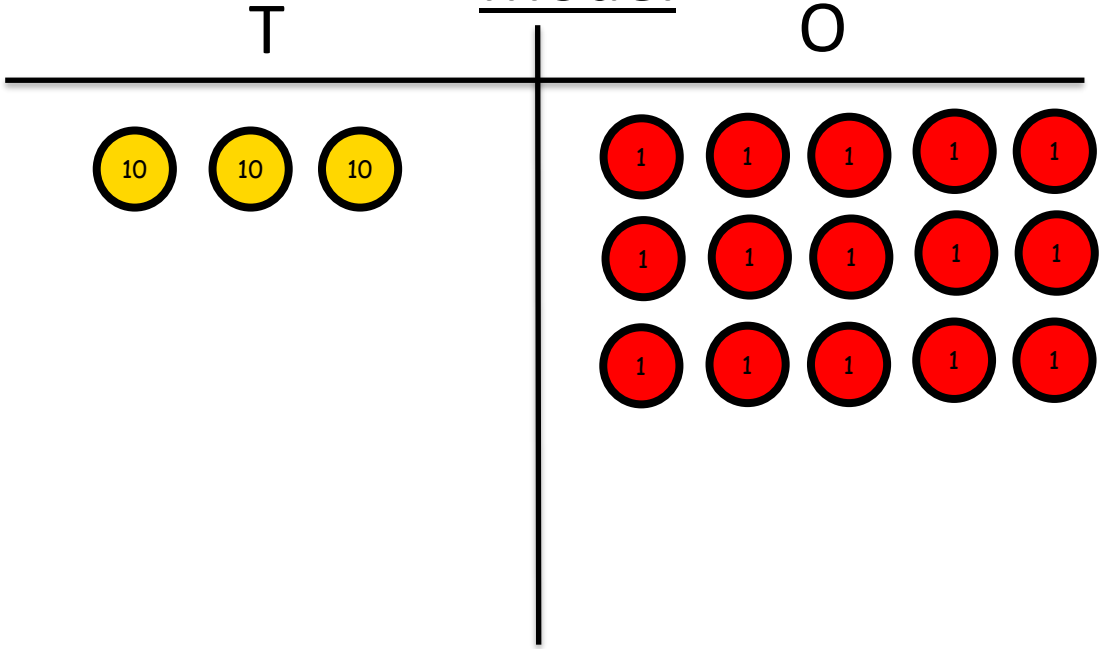
Calculations

14 – 5 = 9

Subtraction- Column method

Solve… 35 - 17 =

Model



Calculations

$$\begin{array}{r} {}^{2}\cancel{3}^{1}5 \\ -17 \\ \hline 18 \end{array}$$

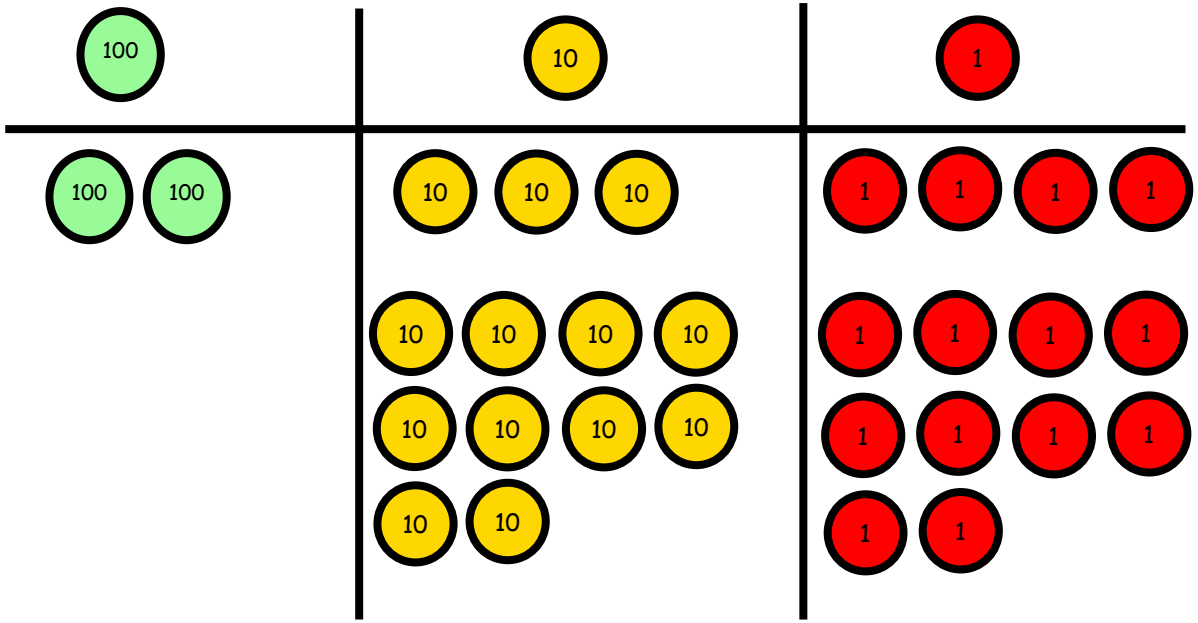
Key vocabulary: **exchange**

Can we exchange any counters?

Subtraction- column method

Solve… 234 – 88=

Model



Calculations

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ 2\ \ 1} \\ 2\not{3}\not{4} \\ -\ \ 88 \\ \hline 146 \end{array}$$

Key vocabulary: **exchange**

Can we exchange any counters?

Pictorial and abstract

As with addition, after lots of experience with the actual counters children should be able to draw the place value counters to help them solve a calculation, crossing out any counters that are to be exchanged. This will then naturally move to working abstractly with the numbers.

Multiplication

Vocabulary

Multiplication	Division
Times Times by Multiply Multiply by Product Product of Lots of Groups of	Share between Share into Split Divide Group Split into

Language of multiplication

$$5 \times 3 = 15$$

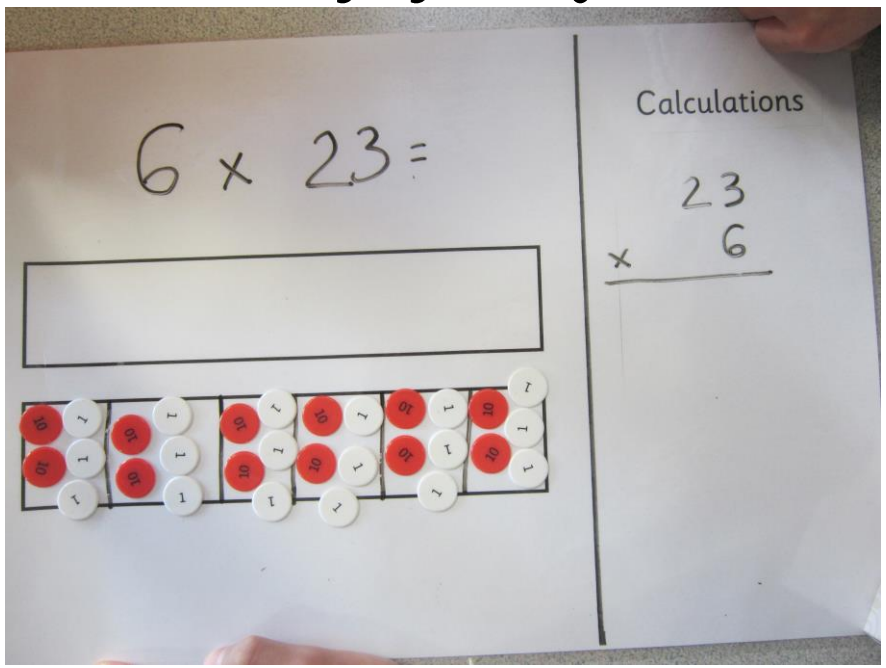
factor

factor

product

Multiplying 2 digit numbers by digit

We often start on a bar model to show 6 lots of 23. This would be done first of all with no crossing over into the tens then we look at exchanging ones for tens.



3 x 6 =

20 x 6 =

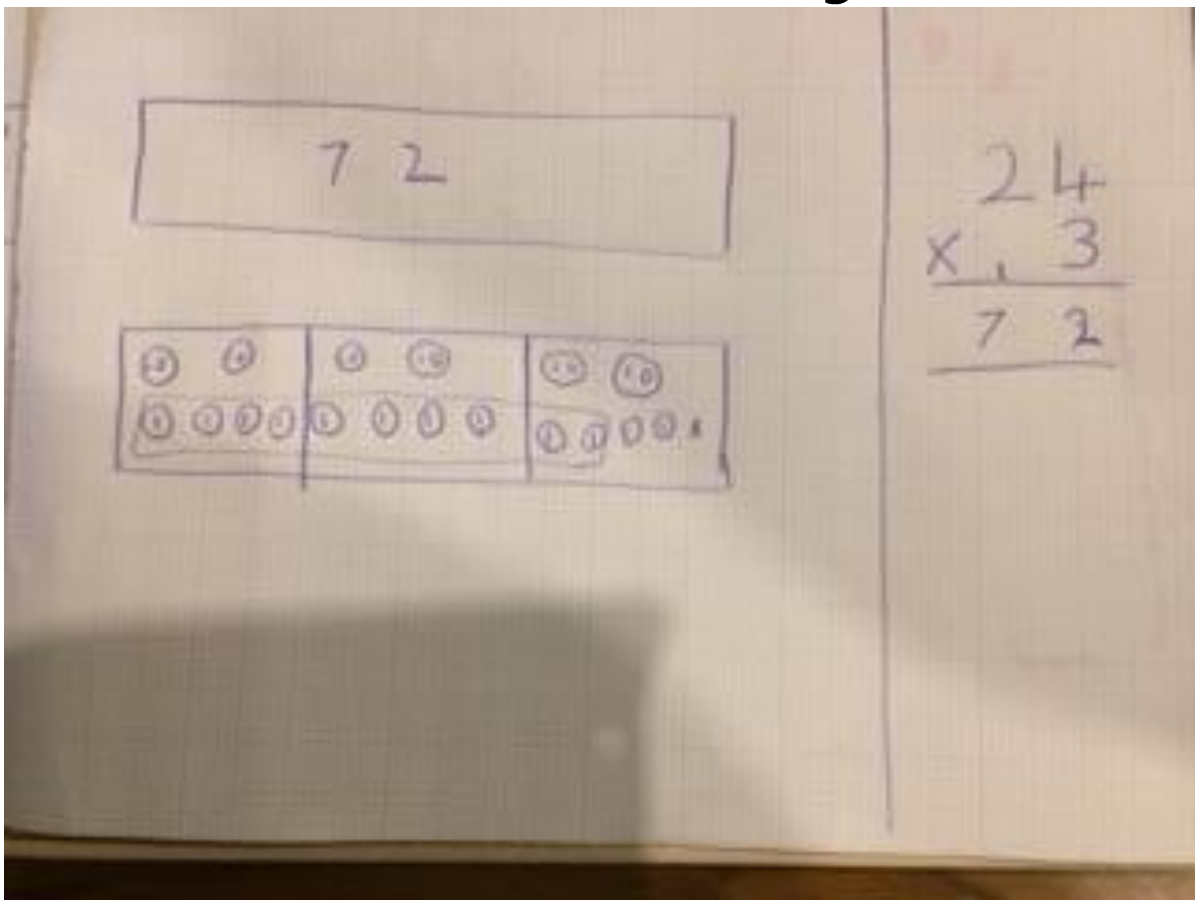
Add the totals together

Place Value Grid

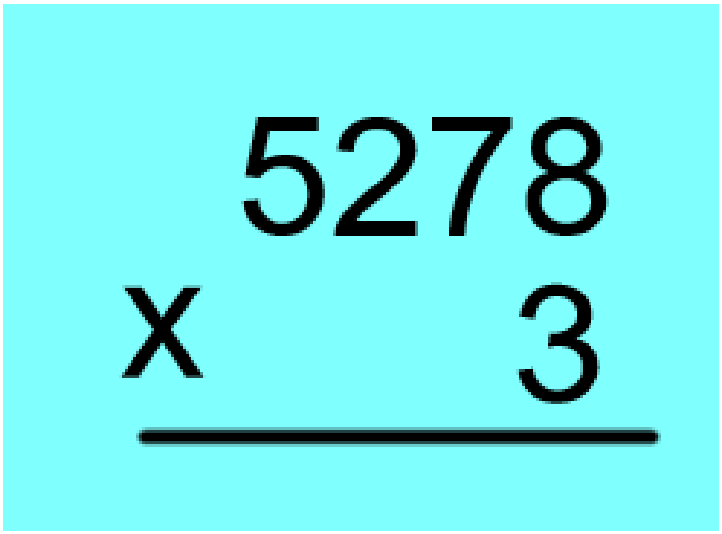
Tens	Ones
10 10	1 1 1
10 10	1 1 1
10 10	1 1 1

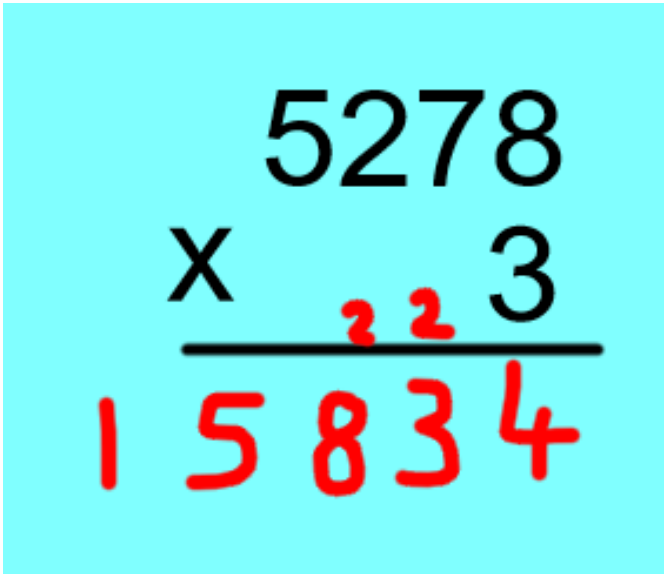
2	3	X	3	=
1	x	3	=	
2	0	x	3	=

Pictorial with 2 digits

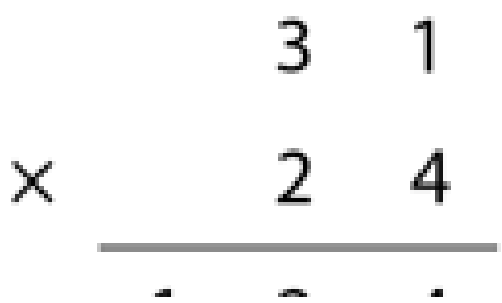


- Eventually, children will use an abstract method to calculate up to 4 digits by one digit including with multiple exchange.
-





Children begin to multiply by 2 digits in Year 5 - partitioning



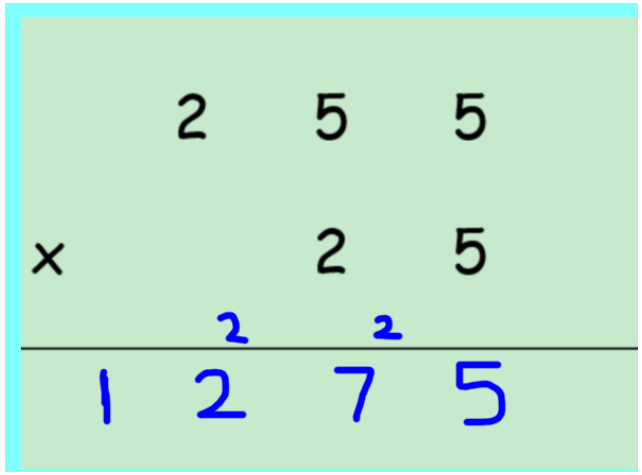
$$\begin{array}{r} 3\ 1 \\ \times\ 2\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3\ 1 \\ \times\ 4 \\ \hline 1\ 2\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3\ 1 \\ \times\ 2\ 0 \\ \hline 6\ 2\ 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6\ 2\ 0 \\ +\ 1\ 2\ 4 \\ \hline 7\ 4\ 4 \\ \hline \end{array}$$

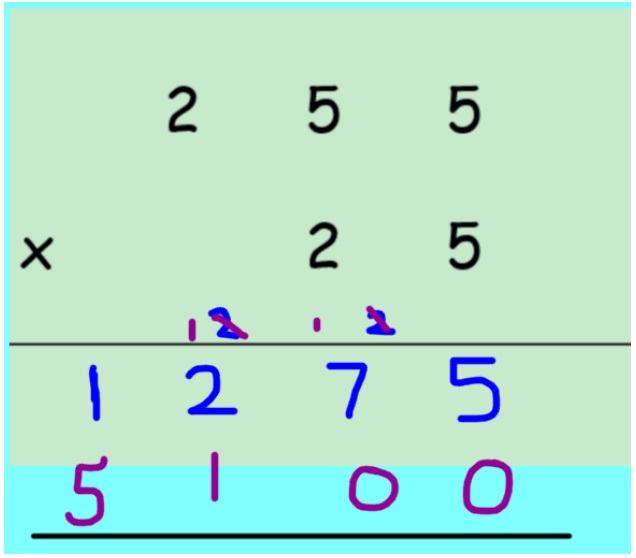
Working abstractly

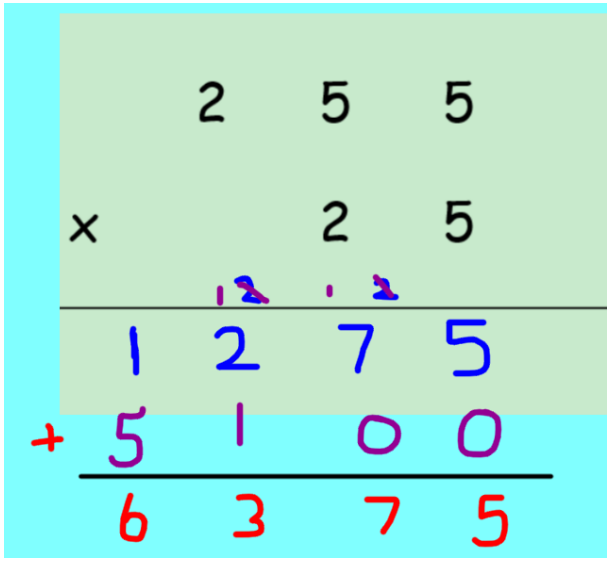


1. Multiply by the ones

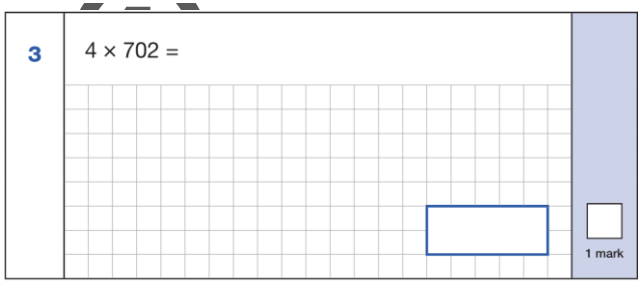
2.Bring down a zero as we are multiplying by tens, the number needs to be ten times the size.

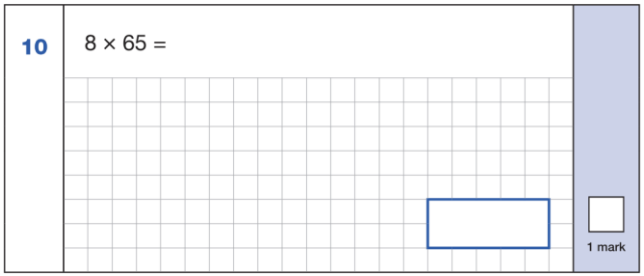
3. Add the 2 parts together.

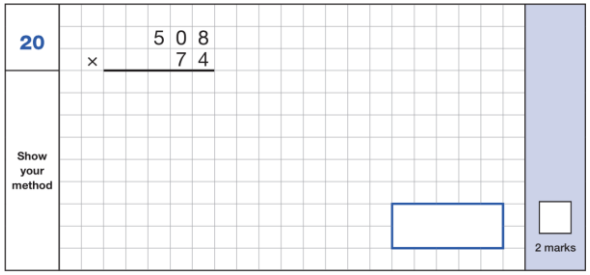


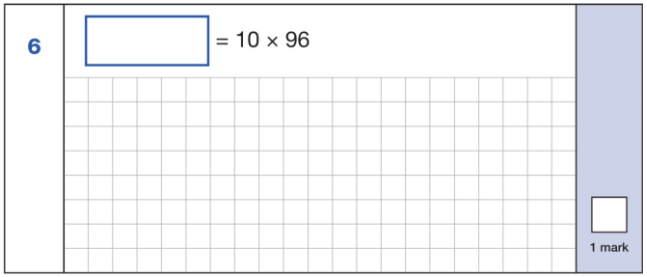


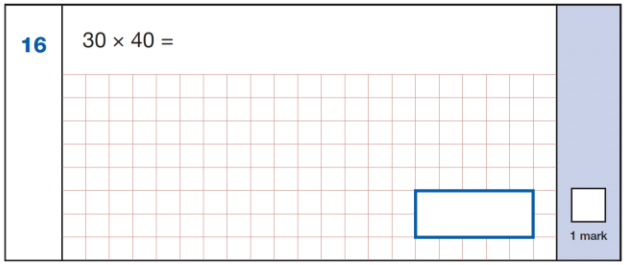
KS2 Arithmetic Paper

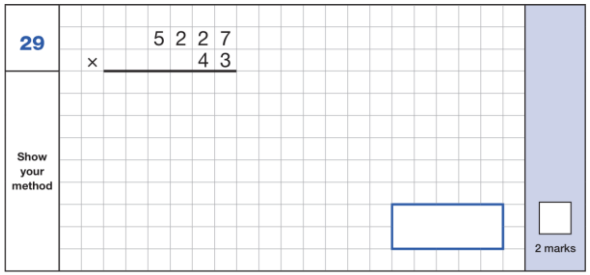












Source: *Q3, 6, 10, 20, 29 2023 KS2 Arithmetic Paper;*
Q16, 2017 KS2 Arithmetic Paper

KS2 Reasoning Paper (1)

13 Write the missing number to make this calculation correct.

$$754 \times 6 + 754 \times 3 = 754 \times \square$$

19 Layla makes jewellery to sell at a school fair.

Each bracelet has **53** beads.

She makes **68** bracelets.

Each necklace has **105** beads.

She makes **34** necklaces.

How many beads does Layla use **altogether**?

20 Write the two missing digits to make this **long multiplication** correct.

	□	2	3	5	
×				□	3
		9	7	0	5
1	6	1	7	5	0
1	7	1	4	5	5

16 A box of 24 chocolate eggs has a mass of **870 grams**.

The empty box has a mass of **30 grams**.

What is the mass of **one** chocolate egg?

Show your method

g

2 marks

Source: *Q3, 2017 KS2 Reasoning Paper 3; Paper 2; Q13, 2023 KS2 Reasoning Paper*

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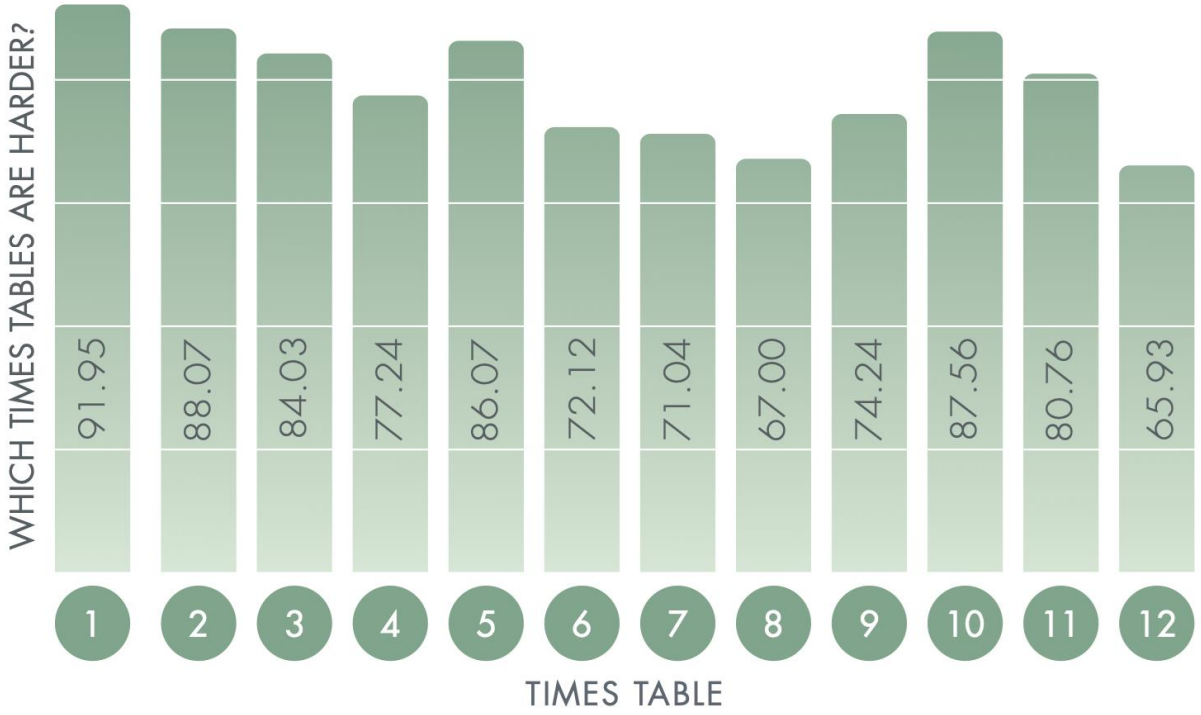
- All of these formal methods for multiplication, ultimately boil down to times table facts. Children who memorise times tables, move very swiftly through column work, as their working memory is not taken up with try to work out 7x8 for instance.

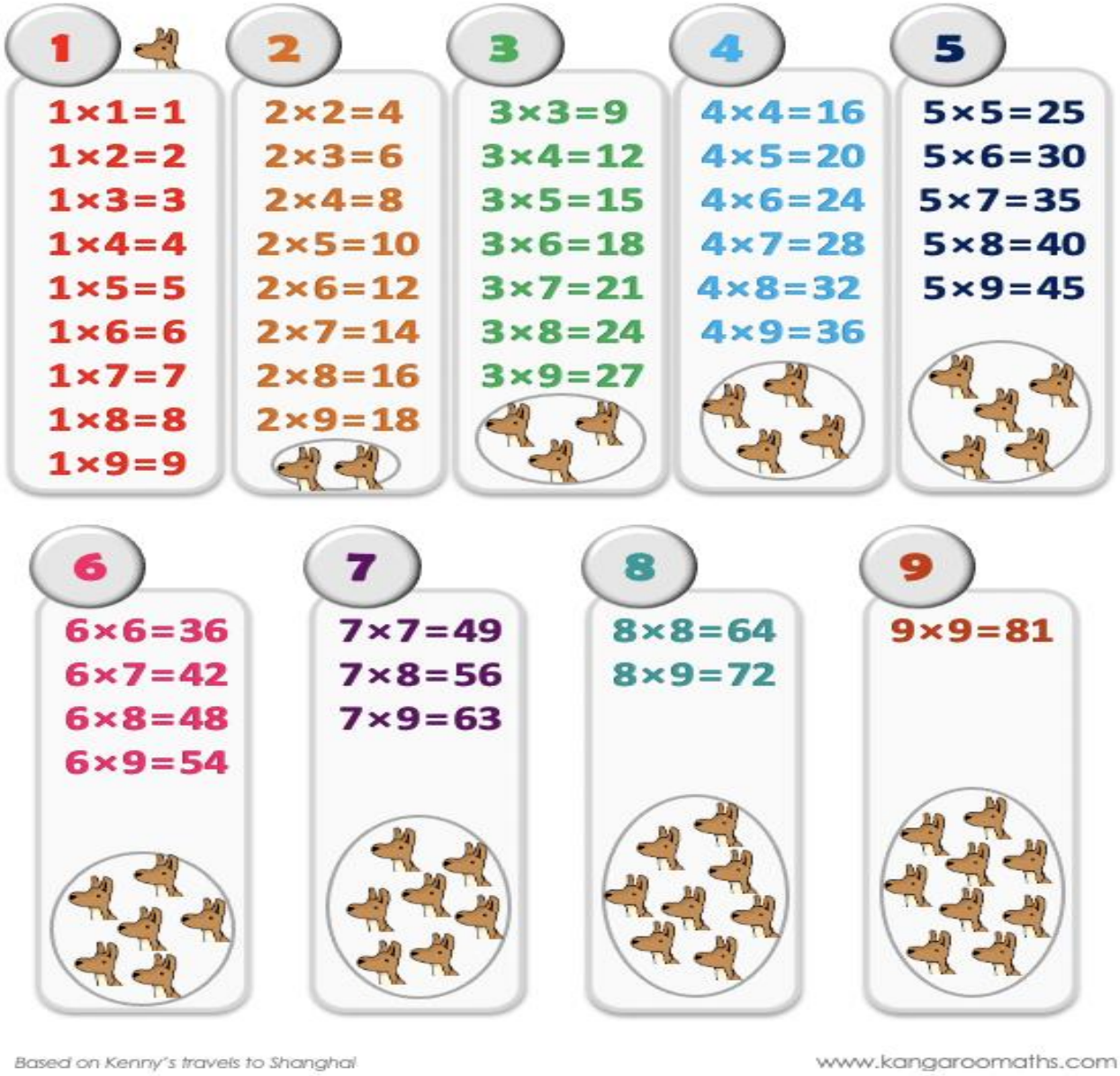
Which multiplication facts do students find tricky?



Source: *https://www.cambridgemaths.org/Images/espresso_1_learning_and_assessing_times_tables.pdf*

Do students need to memorise 144 facts?

1 × 1	1 × 2	1 × 3	1 × 4	1 × 5	1 × 6	1 × 7	1 × 8	1 × 9	1 × 10	1 × 11	1 × 12
2 × 1	2 × 2	2 × 3	2 × 4	2 × 5	2 × 6	2 × 7	2 × 8	2 × 9	2 × 10	2 × 11	2 × 12
3 × 1	3 × 2	3 × 3	3 × 4	3 × 5	3 × 6	3 × 7	3 × 8	3 × 9	3 × 10	3 × 11	3 × 12
4 × 1	4 × 2	4 × 3	4 × 4	4 × 5	4 × 6	4 × 7	4 × 8	4 × 9	4 × 10	4 × 11	4 × 12
5 × 1	5 × 2	5 × 3	5 × 4	5 × 5	5 × 6	5 × 7	5 × 8	5 × 9	5 × 10	5 × 11	5 × 12
6 × 1	6 × 2	6 × 3	6 × 4	6 × 5	6 × 6	6 × 7	6 × 8	6 × 9	6 × 10	6 × 11	6 × 12
7 × 1	7 × 2	7 × 3	7 × 4	7 × 5	7 × 6	7 × 7	7 × 8	7 × 9	7 × 10	7 × 11	7 × 12
8 × 1	8 × 2	8 × 3	8 × 4	8 × 5	8 × 6	8 × 7	8 × 8	8 × 9	8 × 10	8 × 11	8 × 12
9 × 1	9 × 2	9 × 3	9 × 4	9 × 5	9 × 6	9 × 7	9 × 8	9 × 9	9 × 10	9 × 11	9 × 12
10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	10 × 6	10 × 7	10 × 8	10 × 9	10 × 10	10 × 11	10 × 12
11 × 1	11 × 2	11 × 3	11 × 4	11 × 5	11 × 6	11 × 7	11 × 8	11 × 9	11 × 10	11 × 11	11 × 12
12 × 1	12 × 2	12 × 3	12 × 4	12 × 5	12 × 6	12 × 7	12 × 8	12 × 9	12 × 10	12 × 11	12 × 12



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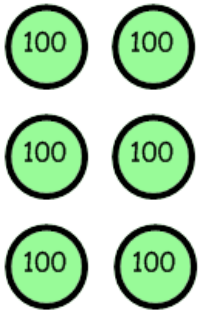
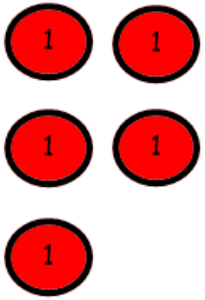
No – potentially, they only need to remember 45 facts. This is presuming children know the tens, and can connect the pattern of 12 x table being x ten plus a double and 11 x table being x 10 plus one more multiple.

Division

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

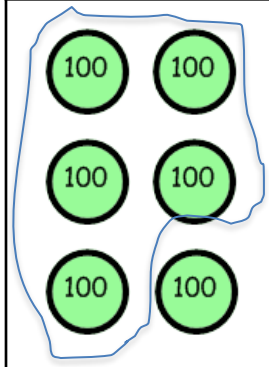
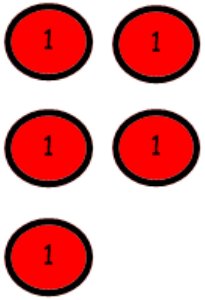
Calculations

$$5\overline{)615}$$

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

Calculations

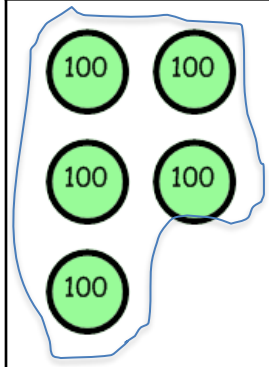
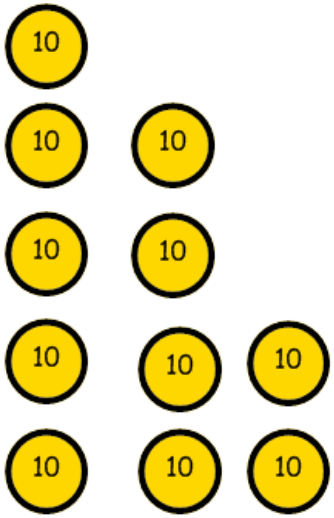
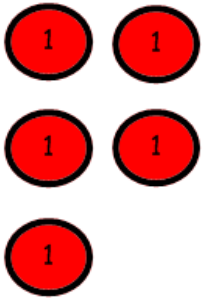
$$5\overline{)615}$$

1

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

Calculations

$$5\overline{)615}$$

$$1$$

Division

Solve… 615 ÷ 5 =

Model

H	T	O
100 100 100 100 100	10 10 10 10 10 10 10 10 10 10	1 1 1 1 1

Calculations

$$5\overline{)615} = 12$$

Division

Solve… 615 ÷ 5 =

Model

H	T	O
100 100 100 100 100	10 10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Calculations

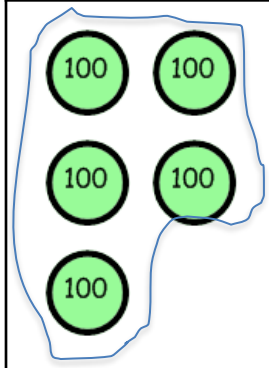
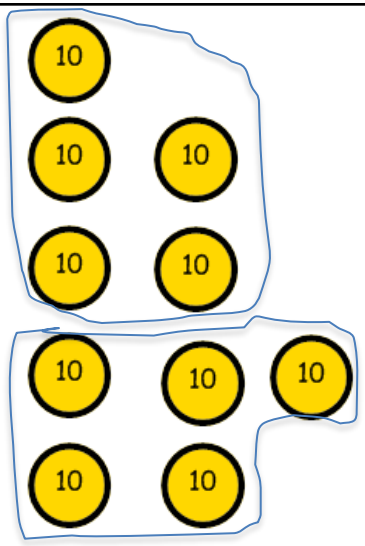
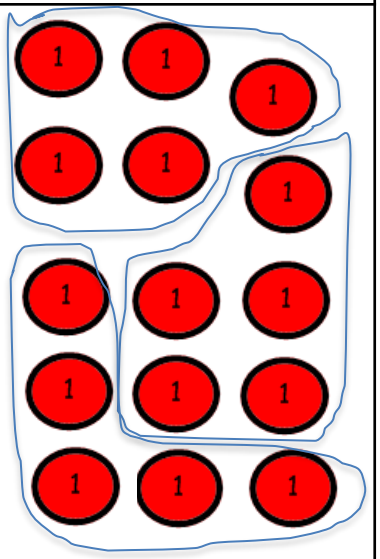
$$5\overline{)615}$$

$$12$$

Division

Solve… 615 ÷ 5 =

Model

H	T	O
		

Calculations

$$5\overline{)615}$$

$$123$$

Long division – Year 6

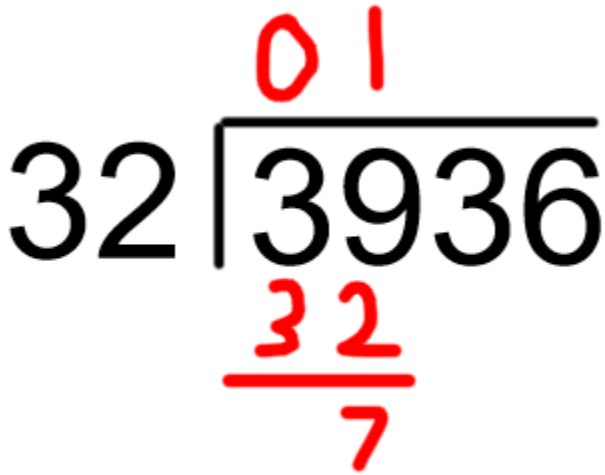
$$32 \overline{)3936}$$

Step 1 Write out the multiples

32
64
96
128
160
192
224

Long division – Year 6

32
64
96
128
160
192
224



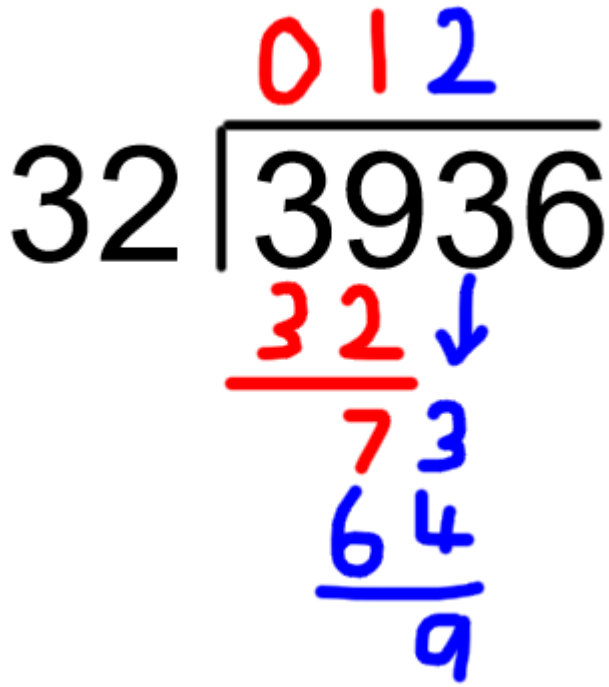
Step 2 Work through as short division, how many 32s in 39

What multiple did I get to? 32.
Write down the multiple and subtract.

This is really the same process as carrying over in short division, but we are writing it underneath.

Long division – Year 6

32
64
96
128
160
192
224

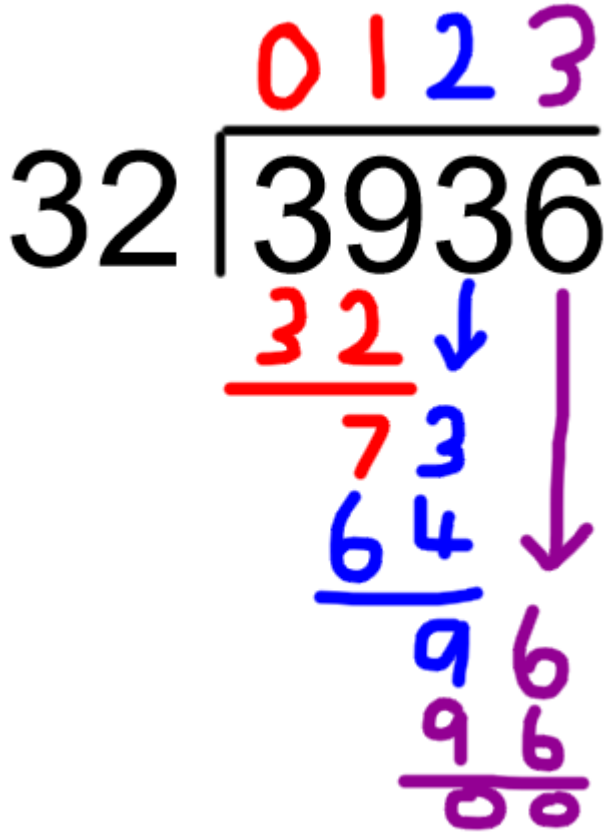


Step 3 Bring down the next digit. SO we are now looking at 73. Repeat the steps. Work through as how many 32s in 73? 2

What multiple did I get to? 64.
Write down the multiple and subtract.

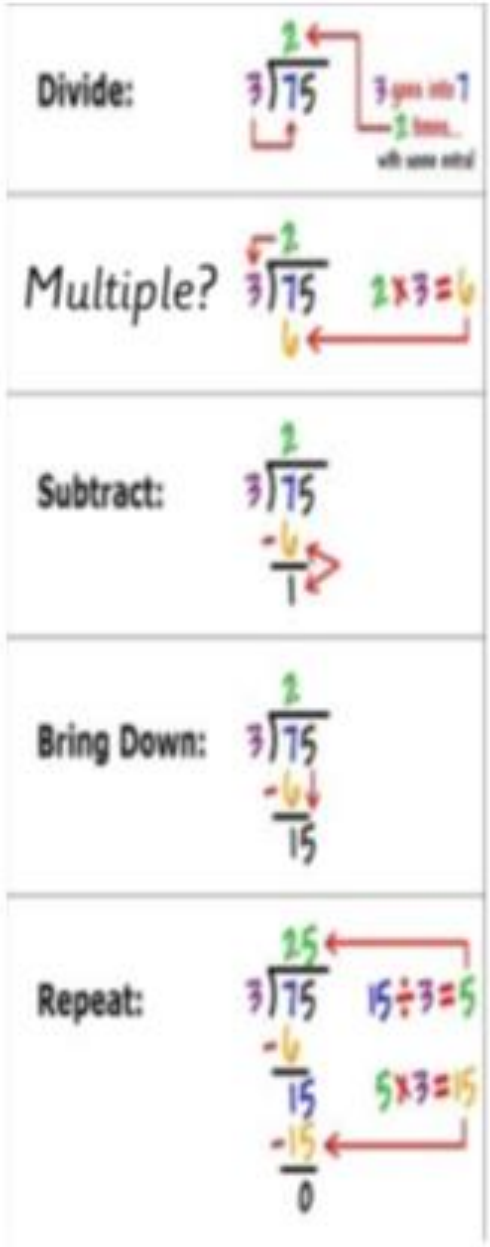
Long division – Year 6

32
64
96
128
160
192
224



Step 4 Bring down the next digit. SO we are now looking at 96. Repeat the steps. Work through as how many 32s in 96? 3

What multiple did I get to? 96.
Write down the multiple and subtract.
There are no remainders.


Divide:
Multiple?
Subtract:
Bring Down:
Repeat:
2 x 3 = 6
15 ÷ 3 = 5
5 x 3 = 15

Thank you for listening.

We hope we have given you a useful insight into using CPA approaches within Mathematics.

If you have any questions please don't hesitate to ask.